Voting is hard!

MITRE STEM Outreach

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What is the goal of this talk?

- **Every 2-4 years there is discussion about if the current voting system is “fair”**
- **We will not tell you if it is or is not fair,**
  - We will introduce concepts necessary for you to decide if it is fair or not
- **This will be done by teaching some methods used:**
  - Borda Counts
  - Instant Runoff
- **Problems with “fairness”:**
  - Arrow’s Theorem
  - Gibbard-Satterthwaite Theorem

The Question

How do we take a collection of opinions, preferences, or choices and combine them in a way that's fair?

In this talk we will show that no system is perfectly fair, in particular when there are more than two choices we can not obtain perfect fairness

Personalized Proportional Voting

- **Each vote consists of two parts:**
  - A choice for personal representative
  - A choice for party preference
- **Parties get additional representatives (from a list) based on the proportion of the party selection they received**

Source: https://commons.wikimedia.org/wiki/File:German_Reichstag.jpg

Ranked Order Voting

- Assume at least 3 choices, and 3 voters
- Each voter ranks the outcomes by preference (most preferred to least)
- The “voting method” takes the lists and produces a list of all the options
- Question: What does it mean for it to be “fair”?

Build a preference order

- **Each participant do the following:**
- **Build an order, of your preference (most preferred to least) of pizza toppings:**
  - A) Anchovies 
  - B) Bell Peppers 
  - C) Canadian Bacon 
  - D) Dried Tomatoes 

Preference Order sample

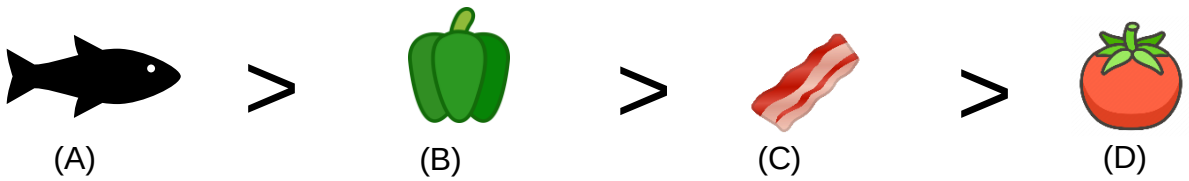
Me	Father	Mother
		
		
		
		

Now, your turn!

- Break up into small groups (3-5).
- Try to assemble a social preference from your pizza topping preferences.
- Record how you decided to build your method.
- Try to keep your method “fair”.

Borda Counts

- **Suppose there are n possible toppings, each voter assigns n points to their favorite topping, $n-1$ to the next, etc.**
- **Each topping is given a score by summing the points for all voters.**
- **The topping with the most points wins!**
  - There are various tie breaking methods.
  - We will use alphabetical preference:



Borda Counts

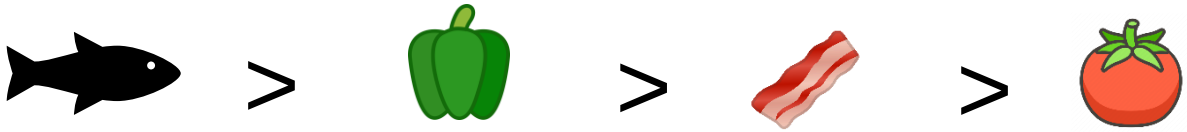
Borda Points	Me	Father	Mother
4 points			
3 points			
2 points			
1 point			

 >  >  > 

8 pts > 8 pts > 7 pts > 7 pts

Instant Runoff

- Assign to each topping the proportion of first place votes received.
- If one topping has at least half, it wins.
- Otherwise (or if building a rank order), delete the topping with the smallest proportion of winners, create a shortened list for each voter and repeat. Use tie breakers if needed.
- The deleted topping then gets the lowest unfilled position in the rank order.
- As with Borda Counts we will use alphabetical order as the tie breaker:



Instant Runoff: Round 1

Me	Father	Mother
		
		
		
		

 has the least (0%) at the top and is removed!

Condorcet's Paradox

Me	Father	Mother
		
		
		

"It is possible to have social preferences that are cyclic, even though individual voter's preferences are not."

This is why we added the alphabetical condition.

Instant Runoff: Round 2

Me	Father	Mother
		
		
		

All of them have equal representation (33%) so

we use alphabetical and remove: 

Instant Runoff: Round 3

Me	Father	Mother
		
		

Since  has a majority (67%) it is a winner!

This gives the following (overall) order:

 >  >  > 

Fairness Conditions…

- **There is no dictator, no one voter always controls the outcome**
- **If every voter prefers one topping to another topping, the group should have the same relative preference**
  - Ex. if everyone has  >  then the group should have this pattern as well
- **The group's relative ranking of two toppings only depends upon the group's individual opinions of the pair of toppings**
  - Ex. if no one changes their relative opinions of  and  , then they shouldn't change in the overall order

Fairness and Borda Counts

- **Consider the Borda Count method, is this "fair" under this definition?**
  - Everyone is treated the same, so there is no dictator!
  - If everyone prefers one topping over another then it will get more points from everyone's list, and so will have more points overall
  - What about the third condition?

Keep Relative  and  but change the rest:

Borda Points	Me	Father	Mother
4 points			
3 points			
2 points			
1 point			

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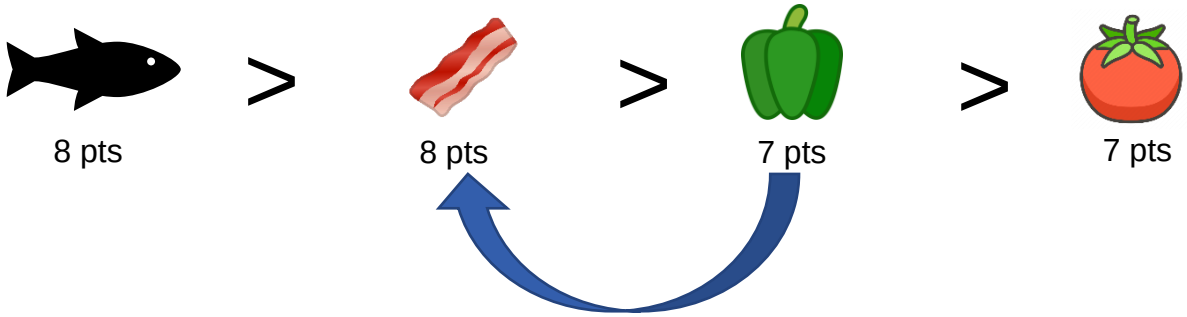


Fairness and Instant Runoff

- **Consider the Instant Runoff method, is this “fair” under this definition?**
  - Everyone is treated the same, so there is no dictator!
  - If everyone prefers one topping over another then it will always have at least as many in the top position.
    - We must ignore the alphabetical condition here!
  - What about the third condition?

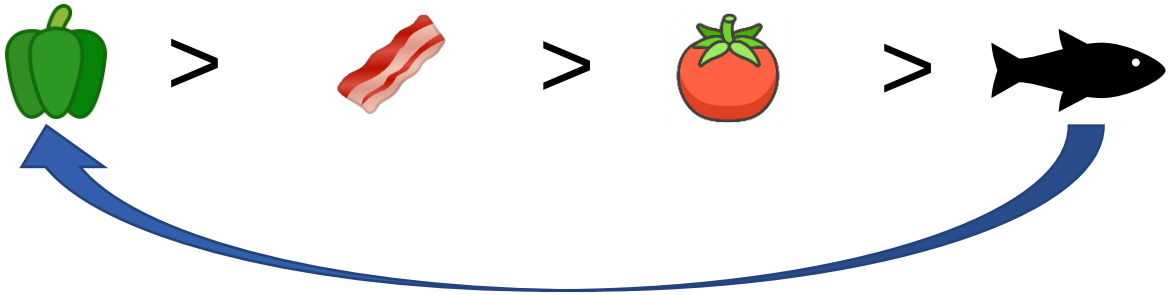
Keep Relative  and  , make one change:

Try this yourselves: Work with your small groups to find one change that changes the over all order to  >  !

Me	Father	Mother
		
		
		
		

Keep Relative  and  , make one change:

Me	Father	Mother
		
		
		
		



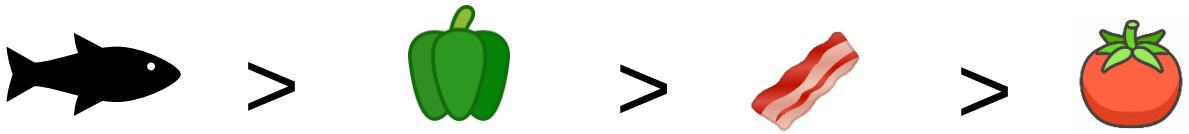
Arrow's Theorem

- **Dr. Kenneth Arrow in his 1951 dissertation:**
- **The three fairness conditions are "incompatible"**
  - That is, we can not have all three of them at the same time
- **This helped him win the Nobel prize in Economics in 1972**

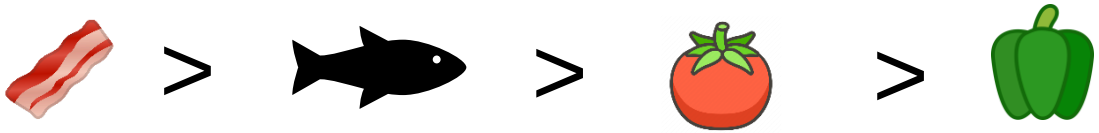
- **What does this mean for us?**
- **If the votes are lists of preferences, it is impossible to have a "fair" voting system, using our definition of fair.**
- **This means that we will need to give up one of the fairness conditions if this is what we want to use!**

Even choosing a voting method is not easy…

- **Different voting method produce wildly different results:**
  - Borda Counts



  - Instant Runoff



  - As a result, even determining which voting system to use could introduce bias against different groups

Your turn again!

- **Maybe we are asking for too much?**
  - Rather than produce a list of preferences from lists of preferences, maybe we can produce 1 winning topping from the lists of preferences?
- **Work with your small groups again, see if rather than making a list you can make a single preference.**
- **That is, each of you have your lists, combine to produce one choice.**

Gibbard-Satterthwaite Theorem

- **Any voting system that chooses a unique topping from a group of preferences must satisfy at least one of the following:**
  - There is a topping which can never win,
  - There is a dictator (who always choses) or,
  - The method is susceptible to “tactical voting”

What is Tactical Voting?

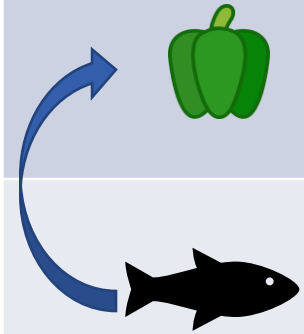
- **Tactical voting is when a voter changes their stated vote, preference, or opinion, in order to obtain a preferred outcome.**
- **That is, if we replace a voter's actual vote, with something else, and the result is the voter gets an outcome they like more, then we have done performed a tactical vote**

An example of Tactical voting:

Me	Father	Mother
		
		
		
		

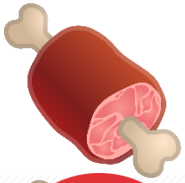
Using Instant Runoff (and just keeping the winner) we had: 

An example of Tactical voting:

Me	Father	Mother
		
		
		
		

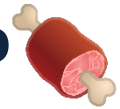
Using Instant Runoff (and just keeping the winner) we had: 

Alternative Voting

- Assume there are exactly two choices, and at least three voters
- We want to choose exactly one outcome
- As before we want this to “fair”
- Take your favorite topping:
-  if your favorite is  or 
-  if it is  or 

May's Theorem [1952]

The only alternative voting system (allowing for abstention votes) which satisfies:

- **The voting method treats each voter identically.**
- **The voting method treats both outcomes the same, in that swapping each vote of  and  swaps the outcome of the vote.**
- **If the group decision was a tie, or  , and a voter changes a vote from  to 0 or  , or from 0 to  , the group decision is  ,**

Is the simple majority

Median Voter Theorem

- **In a simple majority voting system, the outcome will be the outcome preferred by the median voter.**

- **There are some assumptions:**
  - the options can be placed on a one-dimensional spectrum (that is, we put them on a line),
  - each voter has an opinion on the spectrum, and votes for closest choice

Condorcet's Jury Theorem

- In a jury, if each juror has (independent) probability p>.5 of being correct, then the larger the jury, the more likely the correct outcome is chosen, with simple majority.

- This means: If we assume that each voter is more likely than not to choose the "best" candidate, then the more people who vote the better!

In Summary

- **We can not obtain all the conditions that we want to be "fair" in a voting system.**
- **We can get different outcomes using different methods.**
- **Many systems tend to be susceptible to tactical voting.**
- **So, voting is hard. There are many voting systems to choose from, each with its own built-in bias.**
  - Which one should we use? Maybe we should vote on it?

Index / Searchable Terms

- Social Choice Theory
- Arrow's Theorem
- Condorcet's Jury Theorem
- Condorcet's Paradox (The Voting Paradox)
- Gibbard-Satterthwaite Theorem
- May's Theorem
- Median Voter Theorem

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